

# Monopoles in Weinberg-Salam Model

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## Abstract

We present a new type of spherically symmetric monopole and dyon solutions with the magnetic charge  $4\pi/e$  in the standard Weinberg-Salam model. The monopole (and dyon) could be interpreted as a non-trivial hybrid between the abelian Dirac monopole and non-abelian 't Hooft-Polyakov monopole (with an electric charge). We discuss the possible physical implications of the electroweak dyon.

Ever since Dirac [1] has generalized the Maxwell's theory with his magnetic monopole, the monopoles have been a subject of extensive studies. The Abelian monopole has been generalized to the non-Abelian gauge theory by Wu and Yang [2] who constructed a non-Abelian monopole solution in the pure  $SU(2)$  gauge theory, and by 't Hooft and Polyakov [3] who have shown that the  $SU(2)$  gauge theory allows a finite energy monopole solution as a topological soliton in the presence of a triplet scalar source. In the interesting case of the electroweak theory of Weinberg and Salam [4], however, it has generally been believed that there exists no topological monopole of physical interest. The basis for this “non-existence theorem” is, of course, that with the spontaneous symmetry breaking the quotient space  $SU(2) \times U(1)/U(1)_{\text{em}}$  allows no non-trivial second homotopy. This has led many people to conclude that there is no topological structure in the Weinberg-Salam model which can accommodate a magnetic monopole. The purpose of this letter is to show that this conclusion is premature. In the following *we establish the existence of a new type of monopole and dyon solutions in the standard Weinberg-Salam model, and clarify the topological origin of the magnetic charge.* Clearly the new monopole and dyon will have important implications in the phenomenology of the electroweak theory, which can make them very interesting from the physical point of view.

Before we construct the monopole we must understand how one can circumvent the non-existence theorem in the Weinberg-Salam model and obtain the desired solutions. For this it is important to realize that, *with the extra hypercharge  $U(1)$  degrees of freedom, the standard Weinberg-Salam model could be viewed as a gauged  $CP^1$  model in which the (normalized) Higgs doublet plays the role of the  $CP^1$  field.* Viewed as a  $CP^1$  doublet the Higgs field can now admit a topologically non-trivial configuration whose second homotopy is given by  $\pi_2(CP^1) = Z$ . This clears the way for a genuine topological monopole in the Weinberg-Salam model which can be described by a completely regular  $SU(2)$  potential.

To construct the desired solutions we start with the Lagrangian which describes (the bosonic sector of) the standard Weinberg-Salam model

$$\mathcal{L} = -|\hat{D}_\mu \phi|^2 - \frac{\lambda}{2} \left( \phi^\dagger \phi - \frac{\mu^2}{\lambda} \right)^2 - \frac{1}{4} (\mathbf{F}_{\mu\nu})^2 - \frac{1}{4} (G_{\mu\nu})^2,$$

$$\begin{aligned} \hat{D}_\mu \phi &= \left( \partial_\mu - i \frac{g}{2} \boldsymbol{\tau} \cdot \mathbf{A}_\mu - i \frac{g'}{2} B_\mu \right) \phi \\ &= \left( D_\mu - i \frac{g'}{2} B_\mu \right) \phi, \end{aligned} \quad (1)$$

where  $\phi$  is the Higgs doublet,  $\mathbf{F}_{\mu\nu}$  and  $G_{\mu\nu}$  are the gauge fields of  $SU(2)$  and  $U(1)$  with the potentials  $\mathbf{A}_\mu$  and  $B_\mu$ , and  $g$  and  $g'$  are the corresponding coupling constants. Notice that  $D_\mu$  describes the covariant derivative of the  $SU(2)$  subgroup only. From (1) one has the following equations of motion

$$\begin{aligned} \hat{D}_\mu (\hat{D}_\mu \phi) &= \lambda \left( \phi^\dagger \phi - \frac{\mu^2}{\lambda} \right) \phi, \\ D_\mu \mathbf{F}_{\mu\nu} &= -\mathbf{j}_\nu = i \frac{g}{2} \left[ \phi^\dagger \boldsymbol{\tau} (\hat{D}_\nu \phi) - (\hat{D}_\nu \phi)^\dagger \boldsymbol{\tau} \phi \right], \\ \partial_\mu G_{\mu\nu} &= -k_\nu = i \frac{g'}{2} \left[ \phi^\dagger (\hat{D}_\nu \phi) - (\hat{D}_\nu \phi)^\dagger \phi \right]. \end{aligned} \quad (2)$$

Notice that with

$$\begin{aligned} \phi &= \frac{1}{\sqrt{2}} \rho \xi \quad (\rho^2 = 2 \phi^\dagger \phi, \quad \xi^\dagger \xi = 1), \\ \hat{\phi} &= \xi^\dagger \boldsymbol{\tau} \xi, \\ A_\mu &= \hat{\phi} \cdot \mathbf{A}_\mu, \\ C_\mu &= i \xi^\dagger \partial_\mu \xi, \end{aligned} \quad (3)$$

one has

$$\begin{aligned} \mathbf{j}_\mu &= -\frac{g\rho^2}{2} \left[ \frac{g}{2} \mathbf{A}_\mu + \left( \frac{g'}{2} B_\mu + C_\mu \right) \hat{\phi} + \frac{1}{2} \hat{\phi} \times \partial_\mu \hat{\phi} \right], \\ k_\mu &= -\frac{g'\rho^2}{2} \left( \frac{g}{2} A_\mu + \frac{g'}{2} B_\mu + C_\mu \right) = \frac{g'}{g} (\hat{\phi} \cdot \mathbf{j}_\mu). \end{aligned}$$

Now we choose the following static spherically symmetric ansatz

$$\begin{aligned} \rho &= \rho(r), \\ \xi &= i \begin{pmatrix} \sin(\theta/2) e^{-i\varphi} \\ -\cos(\theta/2) \end{pmatrix}, \quad \hat{\phi} = \xi^\dagger \boldsymbol{\tau} \xi = -\hat{r}, \end{aligned}$$

$$\mathbf{A}_\mu = \frac{1}{g}A(r)\partial_\mu t \hat{\phi} + \frac{1}{g}(f(r) - 1)\hat{\phi} \times \partial_\mu \hat{\phi}, \quad (4)$$

$$B_\mu = -\frac{1}{g'}B(r)\partial_\mu t - \frac{1}{g'}(1 - \cos \theta)\partial_\mu \varphi,$$

where  $(t, r, \theta, \varphi)$  are the polar coordinates. Notice that the apparent string singularity along the negative z-axis in  $\xi$  and  $B_\mu$  is a pure gauge artifact which can easily be removed with a hypercharge  $U(1)$  gauge transformation. Indeed one can easily excise the string by making the hypercharge  $U(1)$  bundle non-trivial [2]. So *the above ansatz describes a most general spherically symmetric ansatz of a  $SU(2) \times U(1)$  dyon*. Here we emphasize the importance of the non-trivial  $U(1)$  degrees of freedom to make the ansatz spherically symmetric. Without the extra  $U(1)$  the Higgs doublet does not allow a spherically symmetric ansatz. This is because the spherical symmetry for the gauge field involves the embedding of the radial isotropy group  $SO(2)$  into the gauge group that requires the Higgs field to be invariant under the  $U(1)$  subgroup of  $SU(2)$ . This is possible with a Higgs triplet, but not with a Higgs doublet [5]. In fact, in the absence of the hypercharge  $U(1)$  degrees of freedom, the above ansatz describes the  $SU(2)$  sphaleron which is not spherically symmetric [6]. The situation changes with the inclusion of the extra hypercharge  $U(1)$  in the standard model, which can compensate the action of the  $U(1)$  subgroup of  $SU(2)$  on the Higgs field.

With the spherically symmetric ansatz (2) is reduced to

$$\begin{aligned} \ddot{f} - \frac{f^2 - 1}{r^2}f &= \left( \frac{g^2}{4}\rho^2 - A^2 \right) f, \\ \ddot{\rho} + \frac{2}{r}\dot{\rho} - \frac{1}{2}\frac{f^2}{r^2}\rho &= -\frac{1}{4}(A - B)^2\rho + \lambda \left( \frac{\rho^2}{2} - \frac{\mu^2}{\lambda} \right) \rho, \\ \ddot{A} + \frac{2}{r}\dot{A} - 2\frac{f^2}{r^2}A &= \frac{g^2}{4}\rho^2(A - B), \\ \ddot{B} + \frac{2}{r}\dot{B} &= \frac{g'^2}{4}\rho^2(B - A). \end{aligned} \quad (5)$$

At this point one may wish to compare our dyon with that of Julia and Zee [7], which is obtained from the familiar Lagrangian

$$\mathcal{L}' = -\frac{1}{2}|D_\mu\Phi|^2 - \frac{\lambda}{4}\left(\Phi^2 - \frac{\mu^2}{\lambda}\right)^2 - \frac{1}{4}(\mathbf{F}_{\mu\nu})^2, \quad (6)$$

where  $\Phi$  is the Higgs triplet. With

$$\begin{aligned} \Phi &= -\rho(r)\hat{r}, \\ \mathbf{A}_\mu &= -\frac{1}{g}A(r)\partial_\mu t \hat{r} + \frac{1}{g}(f(r) - 1)\hat{r} \times \partial_\mu \hat{r}, \end{aligned} \quad (7)$$

the Julia-Zee dyon is described by

$$\begin{aligned} \ddot{f} - \frac{f^2 - 1}{r^2}f &= (g^2\rho^2 - A^2)f, \\ \ddot{\rho} + \frac{2}{r}\dot{\rho} - 2\frac{f^2}{r^2}\rho &= \lambda\left(\rho^2 - \frac{\mu^2}{\lambda}\right)\rho, \\ \ddot{A} + \frac{2}{r}\dot{A} - 2\frac{f^2}{r^2}A &= 0. \end{aligned} \quad (8)$$

This shows that there is a remarkable similarity between the two dyons. A closer comparison between the two dyons will be discussed soon.

To integrate (5) one may choose the following boundary condition

$$\begin{aligned} f(0) &= 1, \quad \rho(0) = 0, \quad A(0) = 0, \quad B(0) = b_0, \\ f(\infty) &= 0, \quad \rho(\infty) = \rho_0, \quad A(\infty) = A_0, \quad B(\infty) = B_0, \end{aligned} \quad (9)$$

which guarantees the regularity of the solutions in the  $SU(2)$  sector. With this one can easily show that near the origin one must have

$$\begin{aligned} f &\simeq 1 + \alpha_1 r^2 + \dots, \\ \rho &\simeq \beta_1 r^\delta + \dots, \\ A &\simeq a_1 r + \dots, \\ B &\simeq b_0 + b_1 r + \dots, \end{aligned} \quad (10)$$

where  $\delta = (-1 + \sqrt{3})/2$ . On the other hand asymptotically one must have

$$\begin{aligned} f &\simeq f_1 \exp(-\kappa r) + \dots, \\ \rho &\simeq \rho_0 + \rho_1 \frac{\exp(-\sqrt{2}\mu r)}{r} + \dots, \\ A &\simeq A_0 + \frac{A_1}{r} + \dots, \\ B &\simeq A + B_1 \frac{\exp(-\nu r)}{r} + \dots, \end{aligned} \quad (11)$$

where  $\rho_0 = \sqrt{2/\lambda\mu}$ ,  $\kappa = \sqrt{(g\rho_0)^2/4 - A_0^2}$ , and  $\nu = \sqrt{(g^2 + g'^2)\rho_0}/2$ . Notice that asymptotically  $B(r)$  must approach to  $A(r)$  with an exponential damping.

To determine the possible electric and magnetic charge of the desired solutions we now perform the following gauge transformation on (4)

$$\xi \longrightarrow U\xi = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad (12)$$

$$U = -i \begin{pmatrix} \cos(\theta/2) & \sin(\theta/2)e^{-i\varphi} \\ \sin(\theta/2)e^{i\varphi} & -\cos(\theta/2) \end{pmatrix},$$

and find that in this unitary gauge

$$\mathbf{A}_\mu \longrightarrow \frac{1}{g} \begin{pmatrix} (\sin \varphi \partial_\mu \theta + \sin \theta \cos \varphi \partial_\mu \varphi) f(r) \\ (-\cos \varphi \partial_\mu \theta + \sin \theta \sin \varphi \partial_\mu \varphi) f(r) \\ -A(r) \partial_\mu t - (1 - \cos \theta) \partial_\mu \varphi \end{pmatrix}. \quad (13)$$

In particular we have

$$\begin{aligned} A_\mu^3 &= -\frac{1}{g} A(r) \partial_\mu t - \frac{1}{g} (1 - \cos \theta) \partial_\mu \varphi, \\ B_\mu &= -\frac{1}{g'} B(r) \partial_\mu t - \frac{1}{g'} (1 - \cos \theta) \partial_\mu \varphi. \end{aligned} \quad (14)$$

So expressing the electromagnetic potential  $\mathcal{A}_\mu$  and the neutral potential  $\mathcal{Z}_\mu$  with the Weinberg angle  $\theta_w$

$$\begin{aligned} \begin{pmatrix} \mathcal{A}_\mu \\ \mathcal{Z}_\mu \end{pmatrix} &= \begin{pmatrix} \cos \theta_w & \sin \theta_w \\ -\sin \theta_w & \cos \theta_w \end{pmatrix} \begin{pmatrix} B_\mu \\ A_\mu^3 \end{pmatrix} \\ &= \frac{1}{\sqrt{g^2 + g'^2}} \begin{pmatrix} g & g' \\ -g' & g \end{pmatrix} \begin{pmatrix} B_\mu \\ A_\mu^3 \end{pmatrix}, \end{aligned} \quad (15)$$

we have

$$\begin{aligned} \mathcal{A}_\mu &= -e \left( \frac{1}{g^2} A + \frac{1}{g'^2} B \right) \partial_\mu t - \frac{1}{e} (1 - \cos \theta) \partial_\mu \varphi, \\ \mathcal{Z}_\mu &= \frac{e}{gg'} (B - A) \partial_\mu t, \end{aligned} \quad (16)$$

where  $e$  is the electric charge

$$e = \frac{gg'}{\sqrt{g^2 + g'^2}}.$$

From this we conclude that the desired solutions should carry the following electromagnetic charges

$$\begin{aligned} q_e &= 4\pi e \left[ r^2 \left( \frac{1}{g^2} \dot{A} + \frac{1}{g'^2} \dot{B} \right) \right] \Big|_{r=\infty} = \frac{4\pi}{e} A_1 \\ &= \frac{8\pi}{e} \sin^2 \theta_w \int_0^\infty f^2 A dr, \\ q_m &= \frac{4\pi}{e}. \end{aligned} \tag{17}$$

Also, from the asymptotic condition (11) we conclude that our solutions should not carry any neutral charge,

$$\begin{aligned} \mathcal{Z}_e &= -\frac{4\pi e}{gg'} \left[ r^2 (\dot{B} - \dot{A}) \right] \Big|_{r=\infty} = 0, \\ \mathcal{Z}_m &= 0, \end{aligned} \tag{18}$$

which is what one would have expected.

With this one may now try to find out the desired solutions. Although it appears that (5) does not allow a solution which can be expressed in terms of elementary functions, we find that we can integrate it numerically when  $\kappa$  is positive. The monopole solution with  $A = B = 0$  is shown in Fig.1, and a typical dyon solution is shown in Fig.2. As expected our solutions indeed look very similar to the well-known Prasad-Sommerfield solutions of the Julia-Zee dyon [7]. But, of course, there is a crucial difference. *The new feature here is that our dyon now has a non-trivial  $B - A$ , which represents the neutral  $Z$  boson content of the dyon as shown by (16).* To understand the behavior of the solutions, remember that the mass of the  $W$  and  $Z$  bosons are given by  $M_W = g\rho_0/2$  and  $M_Z = \sqrt{g^2 + g'^2}\rho_0/2$ , and the mass of Higgs boson is given by  $M_H = \sqrt{2}\mu$ . So our result confirms that  $\sqrt{(M_W)^2 - (A_0)^2}$  and  $M_H$  determines the exponential damping of  $f$  and  $\rho$ , and  $M_Z$  determines the exponential damping of  $B - A$ , to their vacuum expectation values asymptotically. These are exactly what one would have expected.

The canonical energy of the dyon is given by

$$\begin{aligned}
E &= E_0 + E_1, \\
E_0 &= \frac{2\pi}{g'^2} M_W \int_0^\infty \frac{dx}{x^2}, \\
E_1 &= \frac{4\pi}{g^2} M_W \int_0^\infty dx \left[ \left( \frac{df}{dx} \right)^2 + \frac{(f^2 - 1)^2}{2x^2} + \frac{2x^2}{g^2} \left( \frac{d}{dx} \frac{A}{\rho_0} \right)^2 + \frac{4f^2}{g^2} \left( \frac{A}{\rho_0} \right)^2 + \frac{2x^2}{g'^2} \left( \frac{d}{dx} \frac{B}{\rho_0} \right)^2 \right. \\
&\quad \left. + 2x^2 \left( \frac{d}{dx} \frac{\rho}{\rho_0} \right)^2 + f^2 \left( \frac{\rho}{\rho_0} \right)^2 + \frac{2x^2}{g^2} \left( \frac{\rho}{\rho_0} \right)^2 \left( \frac{A}{\rho_0} - \frac{B}{\rho_0} \right)^2 \right. \\
&\quad \left. + \frac{2\lambda x^2}{g^2} \left( \left( \frac{\rho}{\rho_0} \right)^2 - 1 \right)^2 \right],
\end{aligned} \tag{19}$$

where  $x = M_W r$  is a dimensionless variable. So the classical energy of the dyon is made of two parts, the infinite part  $E_0$  which solely comes from the point-like hypercharge magnetic monopole and the finite part  $E_1$  which comes from the rest. One might worry about the infinite energy  $E_0$  of the dyon. But from the physical point of view this need not be a serious drawback. The infinite part is still controlled by the weak energy scale  $M_W$ , and could easily be made finite by embedding the  $SU(2) \times U(1)$  to a larger group [8]. It could also be made finite with the introduction of the gravitational interaction [9]. Furthermore it could be treated as the “vacuum” energy when one quantizes the classical dyon. So one could easily subtract or factorize out the infinite part and obtain a finite result in the physical applications of the dyon.

To clarify the topological origin of our dyon it is important to understand the similarity between our dyon and the Julia-Zee dyon. For this notice that the Lagrangian (6) with the Higgs triplet scalar source  $\Phi$ , which allows the Julia-Zee dyon, can be viewed to describe a  $CP^1$  gauge theory. Indeed with the identification

$$\Phi = \rho \hat{\Phi}, \quad \hat{\Phi} = \xi^\dagger \boldsymbol{\tau} \xi, \tag{20}$$

one can easily show that the Lagrangian (6) can be written as

$$\mathcal{L}' = -\frac{1}{2}(\partial_\mu \rho)^2 - \frac{\lambda}{4} \left( \rho^2 - \frac{\mu^2}{\lambda} \right)^2 - 2\rho^2 \left( |D_\mu \xi|^2 - |\xi^\dagger D_\mu \xi|^2 \right) - \frac{1}{4}(\mathbf{F}_{\mu\nu})^2. \tag{21}$$



On the other hand the Lagrangian (1) with (3) can also be written as

$$\begin{aligned}
\mathcal{L} &= -\frac{1}{2}(\partial_\mu \rho)^2 - \frac{\lambda}{2} \left( \frac{1}{2}\rho^2 - \frac{\mu^2}{\lambda} \right)^2 - \frac{1}{2}\rho^2 |\hat{D}_\mu \xi|^2 - \frac{1}{4}(\mathbf{F}_{\mu\nu})^2 - \frac{1}{4}(G_{\mu\nu})^2 \\
&= -\frac{1}{2}(\partial_\mu \rho)^2 - \frac{\lambda}{2} \left( \frac{1}{2}\rho^2 - \frac{\mu^2}{\lambda} \right)^2 - \frac{1}{2}\rho^2 (|D_\mu \xi|^2 - |\xi^\dagger D_\mu \xi|^2) \\
&\quad + \frac{1}{2}\rho^2 \left( \xi^\dagger D_\mu \xi - i\frac{g'}{2}B_\mu \right)^2 - \frac{1}{4}(\mathbf{F}_{\mu\nu})^2 - \frac{1}{4}(G_{\mu\nu})^2.
\end{aligned} \tag{22}$$

Now a simple comparison between (21) and (22) tells that they are almost identical. Indeed the only difference between the two Lagrangians (other than the constant normalization coefficients) is the interaction of the  $U(1)$  gauge field to the other fields through the term  $\rho^2(\xi^\dagger D_\mu \xi - i(g'/2)B_\mu)^2$  in (22), which is invariant under the  $U(1)$  gauge transformation of the  $CP^1$  field  $\xi$ . This shows that the Weinberg-Salam model can also be viewed as a  $CP^1$  gauge theory, in which the Higgs doublet can easily accomodate a non-trivial topology described by  $\pi_2(CP^1) = Z$ . This implies that, with a judicious choice of an ansatz, the Julia-Zee dyon could be extended to a  $SU(2) \times U(1)$  electroweak dyon. More importantly *this shows that our dyon has exactly the same topological origin as the Julia-Zee dyon*,  $\pi_2(CP^1) = Z$ . The new feature here is that when  $\pi_2(CP^1)$  become non-trivial the hypercharge  $U(1)$  should also become non-trivial, which is due to the  $SU(2) \times U(1)$  invariant interaction.

To summarize, we have presented a new type of spherically symmetric electroweak monopole (and dyon) which can be interpreted as a hybrid between the Abelian Dirac monopole and the non-Abelian 't Hooft-Polyakov monopole (with an electric charge) in the  $SU(2) \times U(1)$  gauge theory of Weinberg-Salam. Obviously the monopole must be stable because the magnetic charge cannot evaporate. Of course, the electric charge of our dyon remains a free parameter at the classical level. But, just like the Julia-Zee dyon, the electric charge will be quantized after the quantization of the classical dyon. We close with the following remarks:

1) As we have emphasized, our monopole can be viewed as a electroweak generalization of the Dirac monopole. As obviously it can be also viewed as a electroweak generalization of the 't Hooft-Polyakov monopole. Nevertheless there is an important difference between

these and our monopole. For the known monopoles (of Dirac and 't Hooft-Polyakov) it is well-known that the magnetic charge  $q_m$  obeys the Dirac quantization condition  $q_m = 2\pi n/e$  ( $n$  ; integers), where  $e$  is the minimum electric charge of the theory. In contrast our spherically symmetric monopole carries the minimum magnetic charge  $4\pi/e$ . This suggests that the magnetic charge of the electroweak monopole could obey the Schwinger quantization condition  $q_m = 4\pi n/e$ , rather than the Dirac condition.

2) The fact that our dyon is a electroweak dyon suggests that it should be taken seriously as a realistic object. Assuming its existence, it could have important phenomenological consequences. For example in the leptonic sector the possibility of the helicity changing scattering process at the electroweak scale, or the fermionic zero-mode (zero-energy bound state) in the presence of the monopole, should be carefully re-analyzed. Furthermore in the hadronic sector the monopole catalysis of the proton decay through the Callan-Rubakov effect [10], which should depend on how one embeds the electroweak  $SU(2) \times U(1)$  to a larger group to construct the grand unified theory, should also be re-examined.

3) Finally it must be emphasized that our dyon is different from the one that Nambu and others have discussed before [11], which describes a monopole connected to an anti-monopole by a neutral but real (physical) string. So the Nambu's monopole is not spherically symmetric, and does not approach to the vacuum configuration asymptotically. In contrast our dyon describes a genuine isolated spherically symmetric dyon which is not attached to any physical string, and approaches to the vacuum configuration asymptotically. Topologically, the difference can be traced to the fact that in the Nambu's case the hypercharge  $U(1)$  bundle is trivial but in our case the  $U(1)$  bundle becomes non-trivial.

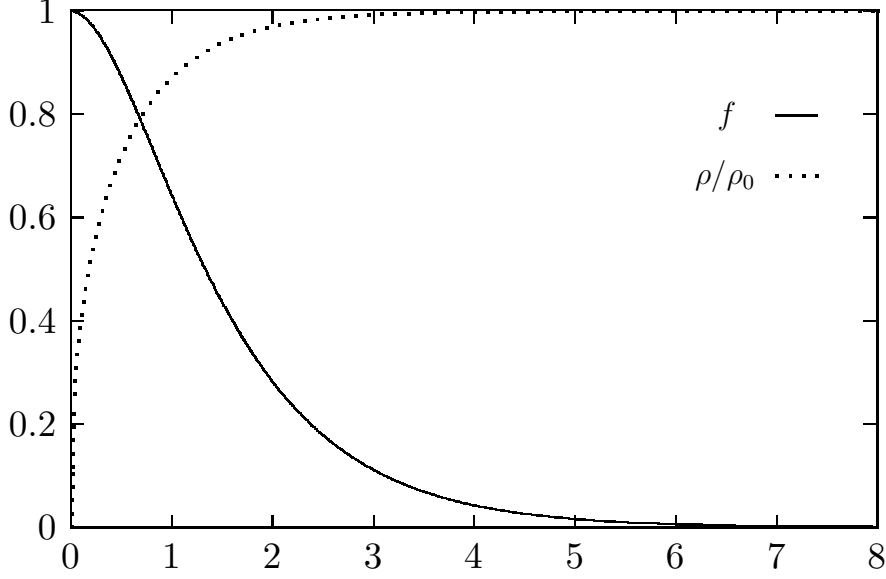
There are many other issues, both mathematical and physical, which need to be addressed in more detail concerning the new dyon. We will discuss these issues in detail in a separate paper.

## ACKNOWLEDGMENTS

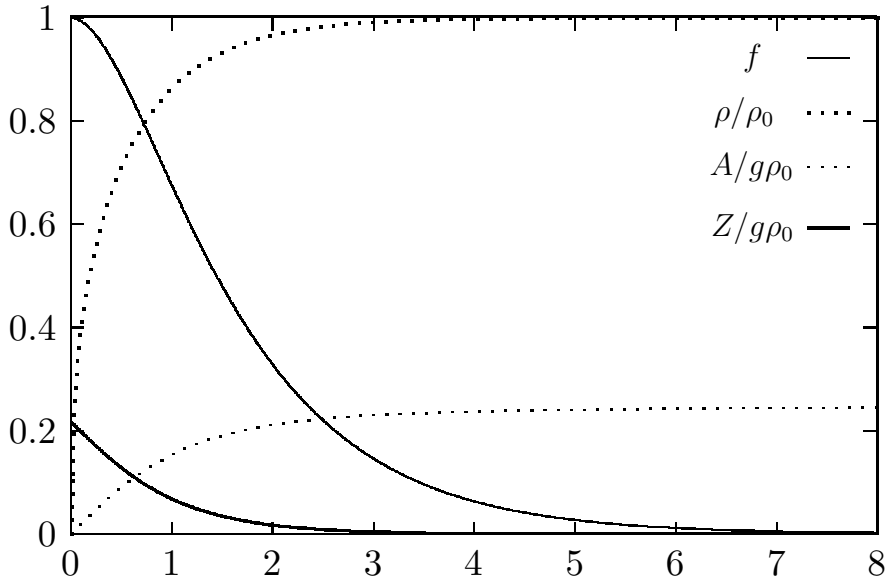
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### Figure Captions

**Fig.1.** The electroweak monopole solution, where we have chosen  $\sin^2 \theta_w = 0.2325$  and  $M_H/M_W = 1$  ( $\lambda/g^2 = 1/4$ ). The plot shows  $f(r)$  and  $\rho(r)$  as functions of dimensionless variable  $x = M_W r$ .



**Fig.2.** A typical electroweak dyon solution, where we have chosen  $A_0 = M_W/2$ . The plot shows  $f(r)$ ,  $\rho(r)$ ,  $A(r)$ , and  $Z(r) = B(r) - A(r)$  as functions of dimensionless variable  $x = M_W r$ .



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